

Systems of Linear and Nonlinear Equations

Supply and Demand

ACTIVITY 13

continued

ACTIVITY 13 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 13-1

Lori was partway up an escalator when her friend Evie realized that she had Lori's keys. Evie, who was still on the ground floor, tossed the keys up to Lori. The function $f(x) = -16x^2 + 25x + 5$ models the height in feet of the keys x seconds after they were thrown. Use this information for Items 1–5.

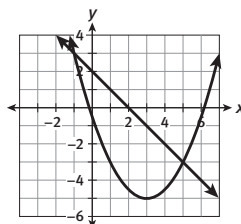
- When the keys are thrown, Lori's hands are 9 ft above ground level and moving upward at a rate of 0.75 ft/s. Write the equation of a function $g(x)$ that gives the height of Lori's hands compared to ground level x seconds after the keys are thrown.
- Write the functions $f(x)$ and $g(x)$ as a system of two equations in two variables. In each equation, let y represent height in feet and x represent time in seconds.
- Graph the system of equations, and use the graph to approximate the solutions of the system.
- How long after the keys are thrown will Lori be able to catch them? Assume that Lori can catch the keys when they are at the same height as her hands. Explain how you determined your answer.
- Explain why your answer to Item 4 is reasonable.

Solve each system by using a graph or table (answers will be approximate).

- $\begin{cases} y = 10 - 2x \\ y = x^2 - 12x + 31 \end{cases}$
- $\begin{cases} y = 5x + 39 \\ y = x^2 + 14x + 52 \end{cases}$
- $\begin{cases} y = -2(x - 3)^2 + 9 \\ y = -4x + 3 \end{cases}$

Use a graph to determine the number of real solutions of each system.

- $\begin{cases} y = 3x^2 + 6x + 4 \\ y = 0.5x + 8 \end{cases}$
- $\begin{cases} y = -2x^2 + 8x - 10 \\ y = -2x + 4 \end{cases}$
- $\begin{cases} y = 24 - 4x \\ y = x^2 - 12x + 40 \end{cases}$
- Which ordered pair is a solution of the system of equations graphed below?



- A. $(-3, 5)$ B. $(-1, 3)$
C. $(2, 0)$ D. $(3, -5)$

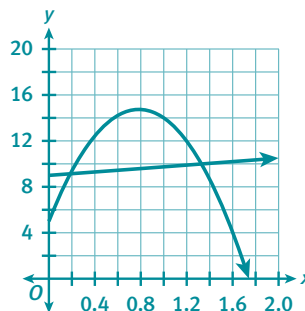
A parallelogram has a height of x cm. The length of its base is 4 cm greater than its height. A triangle has the same height as the parallelogram. The length of the triangle's base is 20 cm.

- Write a system of two equations in two variables that can be used to determine the values of x for which the parallelogram and the triangle have the same area.
- Solve the system by using a graph or table.
- Interpret the solutions of the system in the context of the situation.

ACTIVITY 13 Continued

ACTIVITY PRACTICE

- $g(x) = 0.75x + 9$
- $\begin{cases} y = -16x^2 + 25x + 5 \\ y = 0.75x + 9 \end{cases}$
- Solutions are approximately $(0.2, 9.1)$ and $(1.3, 10.0)$.



- Lori has two chances to catch the keys: about 0.2 s after they are thrown and about 1.3 s after they are thrown. The x -values of the solutions of the system represent how long after the keys are thrown that they will be at the same height as Lori's hands.
- Sample answer: When I substitute 0.2 for x into each equation in the system, I get a value of y that is approximately equal to 9.1. When I substitute 1.3 for x into each equation in the system, I get a value of y that is approximately equal to 10.0. In addition, it makes sense that Lori will have two chances to catch the keys: once when they are on their way up and once when they are on their way down.
- $(3, 4), (7, -4)$
- approximately $(-7.2, 3.0)$, $(-1.8, 30.0)$
- approximately $(0.8, -0.4)$, $(7.2, -25.6)$
- 2 real solutions
- no real solutions
- 1 real solution
- B
- $\begin{cases} y = (x + 4)x \\ y = \frac{1}{2}(20)x \end{cases}$ or equivalent
- $(0, 0), (6, 60)$
- It does not make sense for a parallelogram or a triangle to have a height of 0 cm, so the solution $(0, 0)$ can be ignored. The solution $(6, 60)$ shows that the parallelogram and the triangle have the same area when the height of each is 6 cm. The area of both the parallelogram and the triangle when their height is 6 cm is 60 cm^2 .

ACTIVITY 13 Continued

ACTIVITY PRACTICE

16. $(-3, -10), (0, -7)$
17. $(5, 16)$
18. $(5, -1), (1, -25)$
19. $(-8, 3), (1, -1.5)$
20. $\begin{cases} y = x^2 - 2x - 4 \\ y = -4x - 5 \end{cases}$
21. $(-1, -1)$
22. The solution of the system indicates that the paths of the boats will cross at one point, represented by $(-1, -1)$ on the map.
23. A
24. Sample answer: Use substitution to solve the system. Substitute the expression for y from the first equation into the second equation: $-x^2 + 4x = 3x + 5$. Write the equation in standard form: $0 = x^2 - x + 5$. Use the Quadratic Formula to solve for x : $x = \frac{1}{2} \pm \frac{i\sqrt{19}}{2}$. The values of x are complex conjugates, so the system of equations has no real solutions.
25. $f(x) = 0.75x$
26. $g(x) = 0.015x^2 + 0.15x$
27. $\begin{cases} y = 0.75x \\ y = 0.015x^2 + 0.15x \end{cases}$
28. $(0, 0), (40, 30)$
29. If the length of the longest side is 40 in., the charge for nonglare glass will be the same as the charge for regular glass. This charge will be \$30. Sample explanation: The x -coordinates of the solutions of the system represent lengths for which the charges for the two types of glass will be equal. Because the length of a piece of glass must be greater than 0 in., the solution $(0, 0)$ can be ignored. The solution is $(40, 30)$, meaning when the longest side is 40 in., the charge for both types of glass will be \$30.
30. $\begin{cases} y = 200 + 8x - 0.01x^2 \\ y = 18x \end{cases}$
approximately $(-1020, -18,353)$ and $(20, 353)$; The solutions indicate the number of magnet sets for which Austin's cost of making the magnets will equal his income from selling them. It does not make sense for Austin to make a negative number of magnet sets, so the solution with a negative x -value can be ignored. The solution $(20, 353)$ shows that if Austin makes and sells approximately 20 magnet sets, his cost of making the sets and his income from selling the sets both are about \$353.

ACTIVITY 13

continued

Lesson 13-2

Solve each system algebraically. Check your answers by substituting each solution into one of the original equations. Show your work.

16. $\begin{cases} y = x - 7 \\ y = -x^2 - 2x - 7 \end{cases}$
17. $\begin{cases} y = 2x^2 - 12x + 26 \\ y = 8x - 24 \end{cases}$
18. $\begin{cases} y = -3(x - 4)^2 + 2 \\ y = 6x - 31 \end{cases}$
19. $\begin{cases} y = -0.5x - 1 \\ y = 0.5x^2 + 3x - 5 \end{cases}$

A map of a harbor is laid out on a coordinate grid, with the origin marking a buoy at the center of the harbor. A fishing boat is following a path that can be represented on the map by the equation $y = x^2 - 2x - 4$. A ferry is following a linear path that passes through the points $(-3, 7)$ and $(0, -5)$ when represented on the map. Use this information for Items 20–22.

20. Write a system of equations that can be used to determine whether the paths of the boats will cross.
21. Use substitution to solve the system.
22. Interpret the solution(s) of the system in the context of the situation.
23. How many real solutions does the following system have?

$$\begin{cases} y = -x^2 + 4x \\ y = 3x + 5 \end{cases}$$

- A. none
- B. one
- C. two
- D. infinitely many
24. Explain how you can support your answer to Item 23 algebraically.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

Systems of Linear and Nonlinear Equations

Supply and Demand

A picture-framing company sells two types of glass: regular and nonglare. For a piece of nonglare glass, the charge is equal to the length of the longest side in inches multiplied by the rate \$0.75 per inch. The table shows the charge for several sizes of regular glass.

Charge for Regular Glass

Length of Longest Side (in.)	Charge (\$)
12	3.96
18	7.56
24	12.24

25. Write a linear function $f(x)$ that gives the charge in dollars for a piece of nonglare glass whose longest side measures x inches.
26. Write a quadratic function $g(x)$ that gives the charge in dollars for a piece of regular glass whose longest side measures x inches.
27. Write the functions $f(x)$ and $g(x)$ as a system of equations in terms of y , the charge in dollars for a piece of glass, and x , the length of the longest side in inches.
28. Solve the system by using substitution.
29. For what length will the charge for nonglare glass be the same as the charge for regular glass? What will the charge be? Explain your answers.

MATHEMATICAL PRACTICES

Reason Abstractly and Quantitatively

30. Austin sells sets of magnets online. His cost in dollars of making the magnets is given by $f(x) = 200 + 8x - 0.01x^2$, where x is the number of magnet sets he makes. His income in dollars from selling the magnets is given by $g(x) = 18x$, where x is the number of magnet sets he sells. Write and solve the system, and then explain what the solution(s) mean in the context of the situation.